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# V.H.F. receiving aerials: the use of active elements

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RESEARCH DEPARTMENT

# VHF RECEIVING AERIALS: THE USE OF ACTIVE ELEMENTS

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(RA-60)

Research Department Report No. 1970/31

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SUMMARY

*An array of half-wavelength dipoles forms the most convenient type of aerial for v.h.f. television re-broadcast reception. The principles are well-known by which such an array can be designed to have any specified directivity but re-radiation from the dipoles makes it difficult to implement those principles in practice. This report shows that the difficulty can be overcome by incorporating an amplifier directly across the terminals of each dipole such that the dipoles operate under substantially open-circuit conditions.*

*A suitable design for a practical active dipole is described and, as an example, the directivity that could be achieved using an array of eight such dipoles is given.*

## 1. INTRODUCTION

Many low-power television stations rely for their source of programme upon receiving the signal broadcast from a parent station; this is known as a 're-broadcast link' (r.b.l.). At all receiving sites the field strength is sufficient to produce an adequate signal/noise ratio, but, at several sites, co-channel interference is sometimes comparable in strength with the wanted signal. There is therefore a need for a receiving aerial having good sidelobe suppression, preferably of a type whose directional properties can be suited to the requirements at particular sites.

To be highly directional, an aerial must be large compared to the wavelength. At u.h.f. the wavelengths are such that compact directional aerials can readily be designed but, at v.h.f., size largely dictates the directivity of a practicable array. A convenient configuration for v.h.f. reception comprises an array of half-wavelength dipoles. The overall directivity of such an array is determined by the relative amplitudes and phases of the contributions from all the dipoles forming the array.

If the output from each dipole were to comprise only energy received directly from the incident wave, the principles are straightforward[1,2,3] by which the relative amplitude and phase of the contribution from each dipole could be determined so as to achieve almost any prescribed directivity and to maintain it over a fairly wide band of frequencies. In practice, however, the output from each dipole comprises not only energy received directly from the incident wave but also energy re-radiated from the other dipoles. This re-radiated energy can be allowed for but the procedure is complicated; it becomes very difficult, in general, to achieve a prescribed directivity at a single frequency and virtually impossible to maintain it over a band of frequencies. Consequently, it is customary to combine the outputs from the dipoles in a manner which makes their contributions independent of the re-radiation but this technique can be adopted only if the relative amplitudes and phases of the contributions have a few particular values. This severely restricts the range of directivities that could otherwise be achieved but it is normally accepted as being the only practical solution. Thus re-radiation from the dipoles usually forces the designer to accept a particular directivity because it is feasible in practice rather than because it is the optimum for a given site.

A better solution is to make the re-radiation negligible. The re-radiation from a half-wavelength dipole is very small if it is operated under near-open-circuit conditions but the aerial is then normally very inefficient, a feature that results in a poor signal/noise ratio. This report shows, however, that the principle can be applied to receiving aerials, with only very slight degradation of noise performance, by connecting a high-input-impedance amplifier directly across the terminals of each dipole.[4] Thus, by using what will be termed 'active dipoles,' it becomes practicable to design v.h.f. receiving aerials which have almost any required directivity. At an r.b.l. site where co-channel interference forms the dominant impairment an array of active dipoles would be expected to give a useful improvement. Although the use of active elements as a feature of aerial design has recently received much attention[5,6,7,8,9], it is believed that their use to minimise the effects of mutual couplings in this way is novel.

This report outlines the principles by which the amplifier gains may be chosen in order to achieve a specified directivity. The effect of amplifier input impedance on both the directivity and the noise performance is shown, as also is the variation of directivity with frequency. Finally, a practical design is given for an active dipole which could form part of an active array for v.h.f. television re-broadcast reception.

## 2. DESIGN PRINCIPLES FOR A GIVEN DIRECTIVITY

### 2.1. General

When designing a passive receiving aerial to have a given directivity it is generally more convenient to regard the aerial as transmitting. This is permissible since, by the principle of reciprocity, the directional characteristics are identical.[10] It is shown in Appendix 1 (Section 8.1.) that, with certain reservations, this is true also for active aerials. It is therefore also convenient when designing an active receiving aerial to adopt techniques applicable to transmitting aerials.

The overall directivity of a multi-element array is greatly affected by re-radiation from its elements. It is convenient to regard such re-radiation as being of two types:

1. Re-radiation caused by currents which flow in the element when its terminals are open-circuited

and 2. Re-radiation which occurs when a current flows through the terminals of the element. This is proportional to the current at the terminals.

If re-radiation of the former type is negligible, the design procedure is greatly simplified because the overall radiation from the array is then determined entirely by the relative positions of the elements and the currents flowing at their terminals. Half-wavelength dipoles form very suitable elements because they satisfy this re-radiation requirement and are convenient in practice.

If the dipoles are equi-spaced along a straight line and driven in progressive phase the design procedure is further simplified,[1] particularly for the important case where all the sidelobes are to occur at a specified level[2,3]. For the purpose of this report it is convenient first to summarise this design procedure as applied to transmitting arrays, and then to show how it maybe applied to active receiving arrays. It should be borne in mind that other dipole groupings and phasings may be preferable in particular situations. Only vertically-polarized aerials will be considered in this report; if a horizontally-polarized aerial is required, allowance must be made for the directivity of the individual dipoles.

### 2.2. Design Procedure for Equi-Spaced Collinear Dipoles

Fig. 1 shows, in plan view, a transmitting array of (n+1) vertical dipoles spaced d apart along a straight line. If the currents flowing at the dipole terminals are progressively phased such that the current at the terminals of the rth element is $i_r e^{-jr\theta}$ the field strength at great distance along the bearing $\phi$ (see Fig. 1) will be proportional to:—

$$F = \sum_{r=0}^{n} i_r e^{-jr\psi} \tag{1}$$

where $\psi = \theta - (2\pi d\cos\phi)/\lambda$ (2)

Two important points should be noted here. First, the radiation pattern, when expressed as a function of $\psi$, is determined uniquely by the relative amplitudes of the currents flowing in the dipoles. Second, as $\phi$ varies between 0 and $2\pi$, $\psi$ varies between $(\theta - 2\pi d/\lambda)$ and $(\theta + 2\pi d/\lambda)$. Thus the relative amplitudes of the currents may be chosen so as to achieve a suitable basic pattern and then, quite independently, the relative phases of the currents and the spacing between the dipoles may be chosen so as to use any selected portion of that pattern. This technique is best illustrated by the following example.

Suppose that it is required to design a four-element array. It can be shown that[2,3] the current amplitude distribution 1·0:1·736:1·736:1·0 gives the basic pattern shown in Fig. 2 in which all sidelobes occur at −20 dB relative to the main lobe. Values of $d$ and $\theta$ (Equation 2) may next be chosen so as to use any selected portion of this basic pattern depending upon the type of directivity (e.g. broadside, end-fire etc.) required. For example, if the selected portion lies between the values of $\psi$ equal to $\psi_0$ and $\psi_1$, the phases of the radiating currents and the spacing between the dipoles are defined by:—

$$\theta = (\psi_0 + \psi_1)/2 \tag{3}$$

$$d = \lambda(\psi_0 - \psi_1)/4\pi \tag{4}$$

For a given sidelobe level the beamwidth is least if $\psi_0$ is fixed at the value of $\psi$ at which the field on the flank of the subsidiary main lobe is equal to the sidelobe level (as shown in Fig. 2); $\psi_1$ is then chosen so as to select a portion of the basic pattern according to the type of directional pattern required. The control that the choice of $\psi_1$ has on the directivity is shown in Fig. 3 in which four typical patterns are shown, all of which use the basic pattern of Fig. 2 but which diffef with regard to the choice of $\psi_1$.

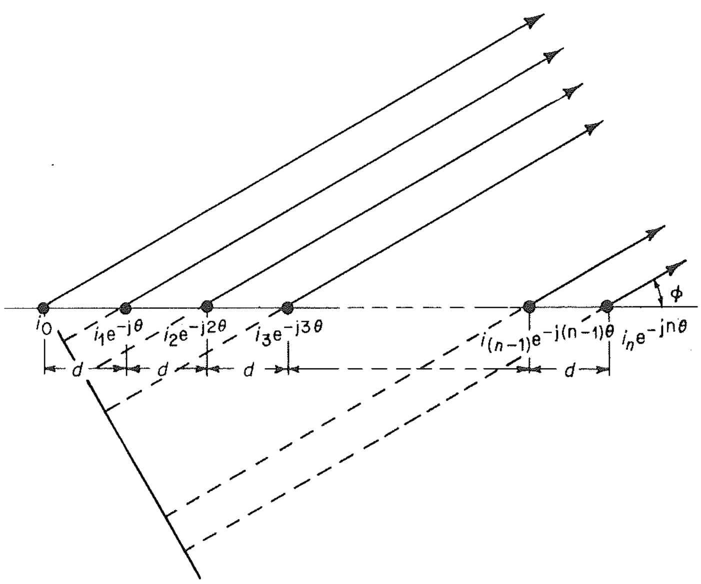


*Fig. 1 - A collinear array of equi-spaced dipoles*

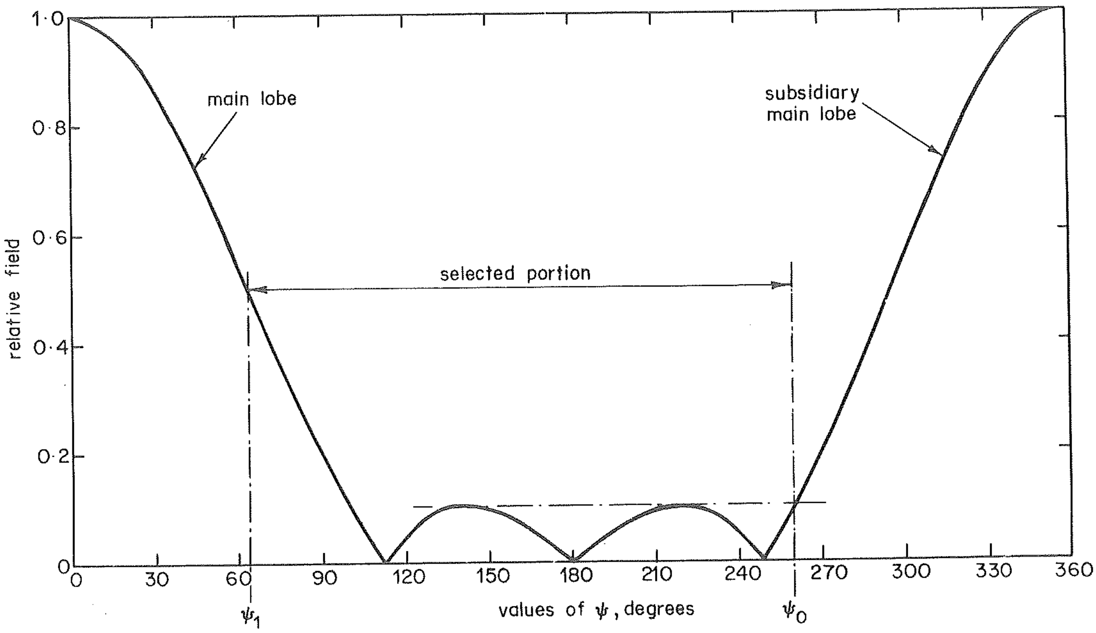


*Fig. 2 • Basic radiation pattern for four-element array*

The basic pattern is shown dotted and the portion selected for each typical pattern is shown by a full line. In Fig. 3(*a*) $\psi_1 = -\psi_0$; all the radiating currents are co-phased and the pattern is that of a broadside array. In Fig. 3(*b*) $\psi_1$ has a value intermediate between 0 and $-\psi_0$; the pattern is that of a slewed array. In Fig. 3(*c*) $\psi_1 = 0$; the pattern is that of a simple end-fire array in which the field contributions from all the dipoles add in phase only along the bearing of the main lobe. In Fig. 3(*d*) $\psi_1$ has a value intermediate between 0 and $\psi_0$ (equal, in fact, to the value shown in Fig. 2); the pattern is that of an over-phased end-fire array in which the field contributions do not add in phase along any bearing. It should be noted that over-phasing in this way narrows the main lobe but worsens the sidelobe level.

The broadside pattern of Fig. 3(*a*) is unlikely to have any application for a re-broadcast receiving aerial. The slewed pattern of Fig. 3(*b*) would be useful at a receiving site where a high degree of suppression is required over a wide range of bearings but where a small range of bearings exists over which no suppression is required.[11] The over-phased pattern of Fig. 3(*d*) would be useful if a narrow main lobe were the prime requirement, but it has two disadvantages. In the first place, the sidelobe level varies with frequency and is greater than that of the simple end-fire aerial. Secondly, small errors in the amplitude and phase of the current distribution have more effect on the pattern of an over-phased aerial than on that of a simple end-fire aerial. In general the most useful pattern for rebroadcast reception is likely to be that of Fig. 3(*c*), or one intermediate between those of Figs. 3(*c*) and 3(*d*).

## 3. THE EFFECT OF AMPLIFIER INPUT IMPEDANCE

### 3.1. General

Fig. 4 shows the type of active receiving array considered in this report. It comprises n vertical dipoles equi-spaced along a straight line, each dipole having a pre-amplifier with voltage gain g, constant group delay over the frequency band and input impedance $z$, connected directly across its terminals. The output of the rth pre-amplifier is connected to a main amplifier of voltage gain $a_r$ through a cable of electrical length $r\theta$ and thence to a common combining point. This is a convenient practical arrangement because, since the *relative* gains of the amplifiers must not vary with changes of atmospheric temperature, it is best to make all the pre-amplifiers identical.

The circuit of a suitable pre-amplifier, comprising a cascode arrangement with provision for feeding the d.c. power through the r.f. output cable, is shown in Fig. 5. The main amplifiers should be designed to have equal group delays; a suitable design for them and for the combining amplifier is given in Reference 11. It is assumed that the cables operate under matched conditions. The effective voltage gain of the amplifiers and cable associated with the rth dipole may therefore be taken as proportional to $a_r e^{-jr\theta}$. The following shows how the design procedure outlined in Section 2.2. may be used to determine suitable values for $a_r$ and $r\theta$ such that the array shall have a prescribed directivity.

When a wave is incident on the array, e.m.f.'s are induced in each of the dipoles, both by the incident wave itself and by re-radiation from the other dipoles.

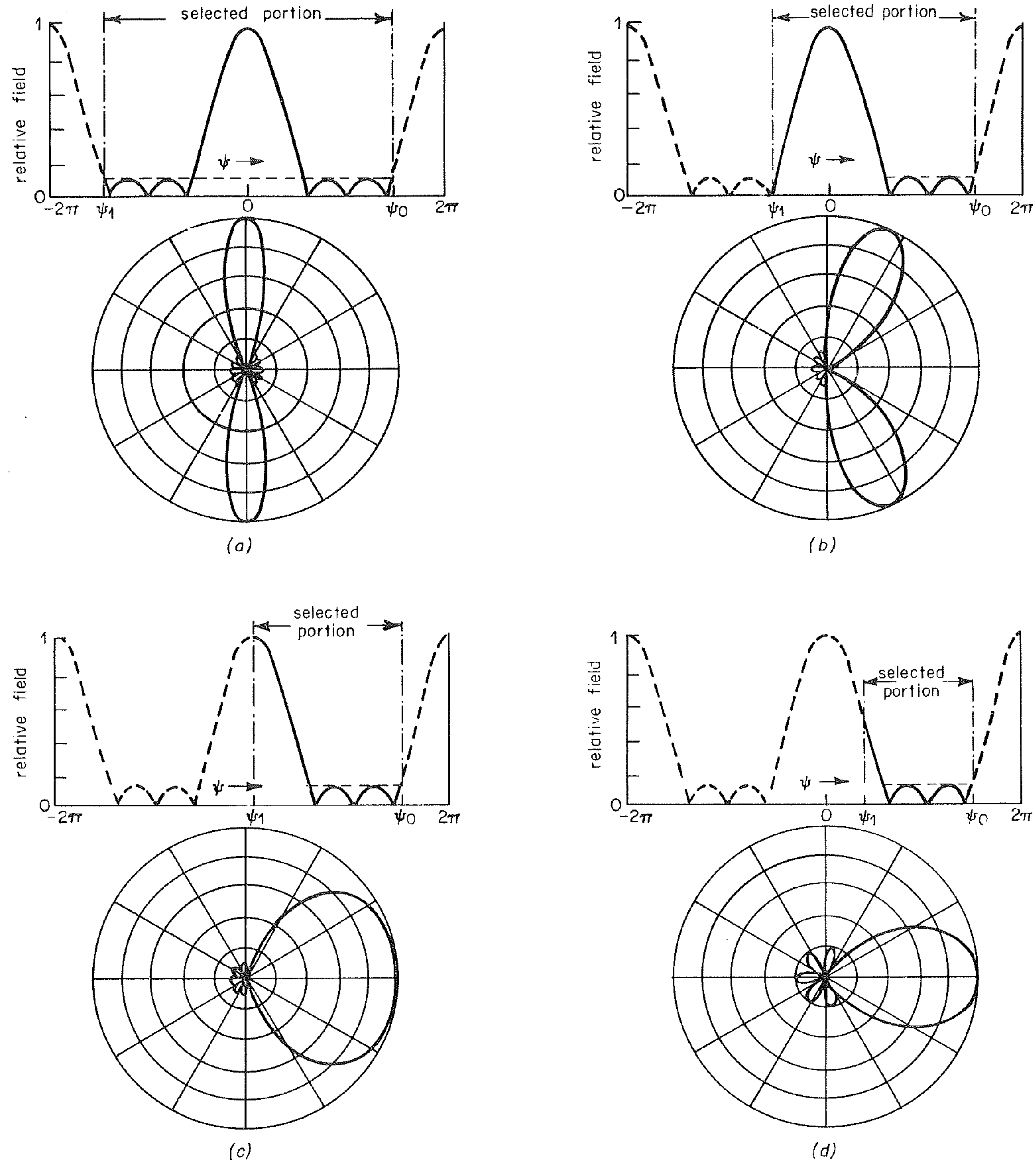


*Fig. 3 - Effect of range of phase variations on directivity*

If re-radiation were negligible, $a_r$ and $r\theta$ would be made equal to the relative amplitudes and phases of the currents as determined in Section 2.2. Provided that re-radiation is negligible when the terminals of the dipoles are open-circuit (this is shown in Section 4 to be the case), the effect of re-radiation which occurs when currents flow through the dipole terminals could be allowed for by suitably modifying the values of $a_r$ and $r\theta$, but such re-radiation is undesirable for two reasons: first, the re-radiation is not usually known with sufficient accuracy and, second, it would be difficult to allow for its variation with frequency. Such re-radiation can be made small by making the input impedance of the pre-amplifiers, $z$, large but the signal/noise ratio worsens as $z$ is increased. Thus $z$ should have a compromise value which is large enough to make the directivity of the array sufficiently independent of re-radiation yet not large enough to worsen the noise level unduly. Sections 3.2. and 3.3. show how the signal/noise ratio and the directivity vary with $z$; Section 3.4. shows how the directivity varies with frequency when $z$ is given a typical compromise value.

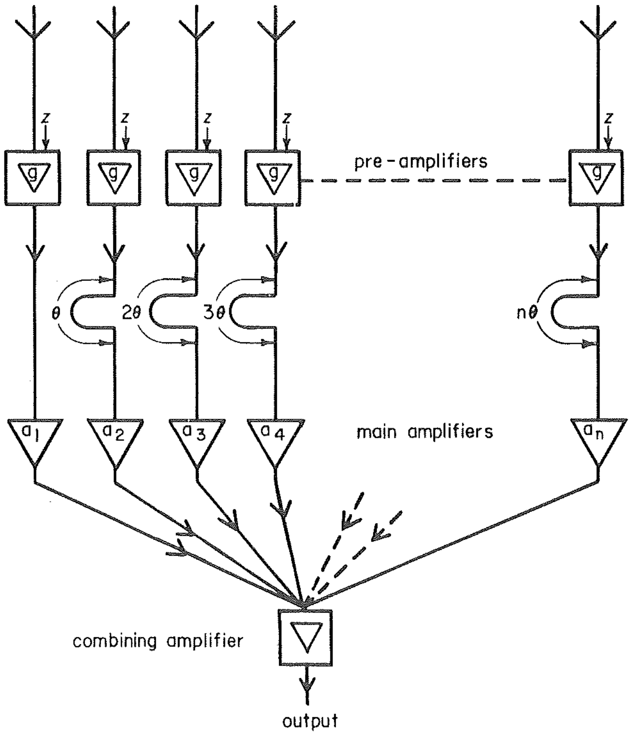


*Fig. 4 - Schematic diagram of active receiving array*

### 3.2. The Effect of Amplifier Impedance on Signal/Noise Ratio

The input conductance of the pre-amplifier shown in Fig. 5, and its noise factor when used with a 60 ohm source, was measured with various values of emitter resistance $R_3$. The results are shown in Fig. 6($a$) and, from those results, the graph of Fig. 6($b$) may be constructed to show the relation between the noise factor and the input conductance. The noise factor quoted by the manufacturers for the input transistor of the pre-amplifier is 2 dB when used under optimally-matched conditions. It is clear from Fig. 6($b$) that the noise factor is degraded by less than 2 dB if, when used with a 60 ohm source, the input conductance of the pre-amplifier lies within the range 0·5 to 2·0 mmho. It is shown in Section 5 that, when the effect of galactic noise is considered, this negligibly affects the effective signal/noise ratio.

### 3.3. The Effect of Amplifier Impedance on Directivity

The equivalent circuit of the active array of Fig. 4 is shown in Fig. 7. An amplifier with input impedance $z$ is connected across each dipole, of self impedance $Z_{rr}$. An e.m.f., $\eta f_r(\phi)$, is induced in the rth dipole which is proportional to the field strength of the incident wave, $\eta$, as a function of its angle of arrival, $\phi$.* In addition, other e.m.f.'s are induced which are proportional both to the currents, $i_m$, flowing through the terminals of the other dipoles and to the mutual impedances, $Z_{rm}$, between the dipoles. The latter are the e.m.f.'s induced by the re-radiation which occurs when currents flow through the dipole terminals (see Sections 2.1. and 3.1.). As shown in Appendix 1 (Section 8.1.), the overall directivity of the array may be calculated, for each value of $\phi$, by multiplying the resultant voltage across each amplifier input by the complex amplifier gain, $a_r e^{-ir\theta}$, and adding the several contributions to form the combined output. The mutual impedances (and their variation with frequency) are not under the designer's control but their effect may be made negligible by using a sufficiently high value of $z$; a sufficiently high value may be judged for a particular array by comparing the directivities which are calculated assuming various finite values of $z$ with that calculated when $z$ is

* $\eta$ is the field strength of the incident wave in the absence of the aerial. If the aerial elements were of a type which caused significant re-radiation when their terminals were open-circuited, the effect of such re-radiation would be included in $f_r(\phi)$. As shown in Section 4, however, the dipoles described in this report cause negligible re-radiation when their terminals are open-circuited; for this application, therefore, $f_r(\phi)$ is a function of only the relative positions of the dipoles.

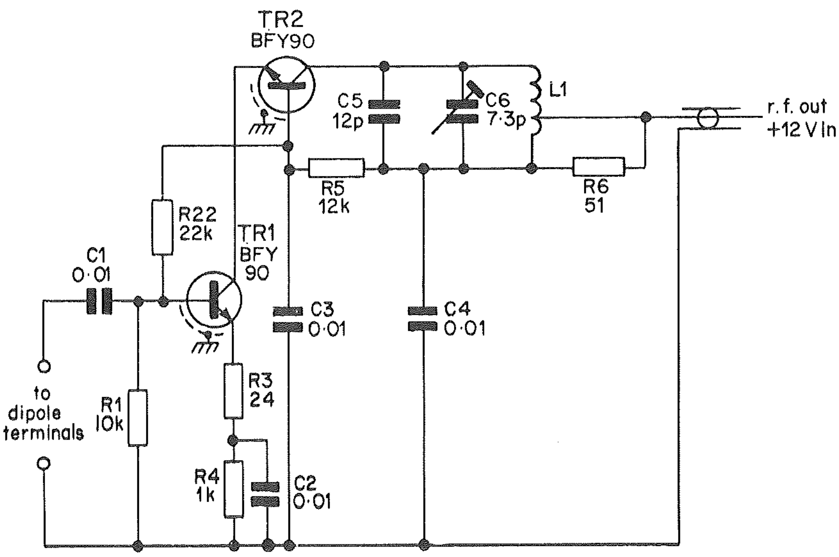


*Fig. 5 - Circuit diagram of pre-amplifier*

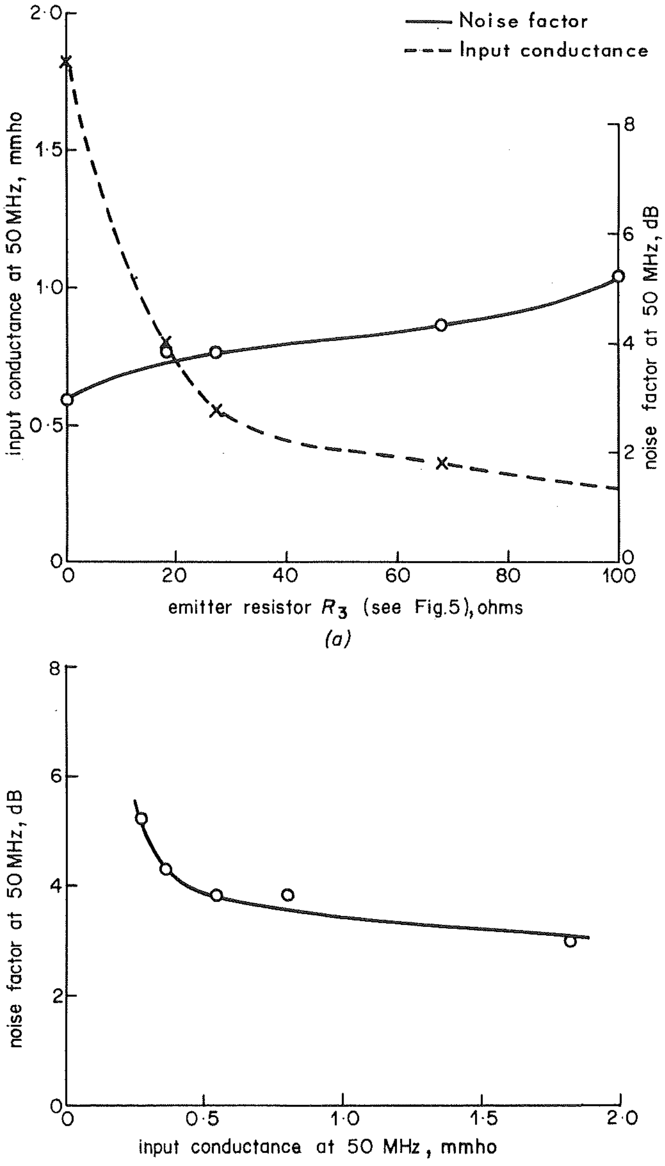


*Fig. 6 • Variation of noise factor and input conductance as a function of emitter resistance $R_3$ (See Fig. 5)*

($a$) Input conductance and noise factor as a function of emitter resistance

($b$) Noise factor as a function of input conductance

infinite. A typical comparison is given in Fig. 8 in which the directivities of an eight-dipole array are shown for three different values of $z$. The array is designed to give an end-fire pattern of the type shown in Fig. 3($c$) having, when $z$ is infinite, equi-level side-lobes at –35 dB relative to the main lobe. For the purpose of Fig. 8 it was assumed that the self-impedance of each dipole ($Z_{rr}$) was equal to (70 + j 0) ohms. It was also assumed that the input circuit of each amplifier was tuned such that its input impedance was purely resistive, with the values given in Fig. 8. These, and all other quantities involved in the calculations, are given in Appendix 2 (Section 8.2.).

It is evident that to use amplifiers with input resistance about five to ten times the dipole self impedance would make the effect of mutual impedance sufficiently small for most practical purposes.

### 3.4. The Variation of Directivity over a Band of Frequencies

The directional characteristics of the array will vary over a band of frequencies due to various factors, some of which are fundamental (e.g. the varying separation, in wavelengths, between the dipoles) and others which are not (e.g. the variation of dipole impedance ($Z_{rr}$), mutual impedance ($Z_{rm}$) and amplifier input impedance ($z$)). The varying separation between the dipoles has least effect if $d$ and $\theta$ (see Section 2.2.) are calculated from equations (3) and (4) at the highest working frequency. The effect of variation of dipole impedance and mutual impedance can be made small by making the input impedance of the amplifiers sufficiently large. The overall variation of directivity in a typical case is shown by the following practical example.

Suppose the input conductance of the pre-amplifiers is 2·0 mmho and that the total capacitance across the terminals of each dipole (e.g. due to the pre-amplifier input circuit plus stray capacitance in the dipole mounting) is 40 pF. This corresponds to an effective pre-amplifier input admittance of (2·0 + j12·65) mmho at 50 MHz ($G$ + j$B$ in the insert of Fig.

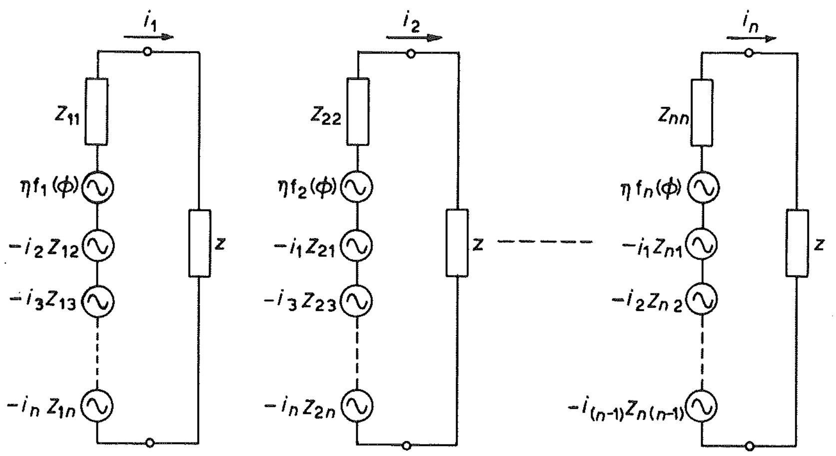


*Fig. 7 • Equivalent circuit of active receiving array*

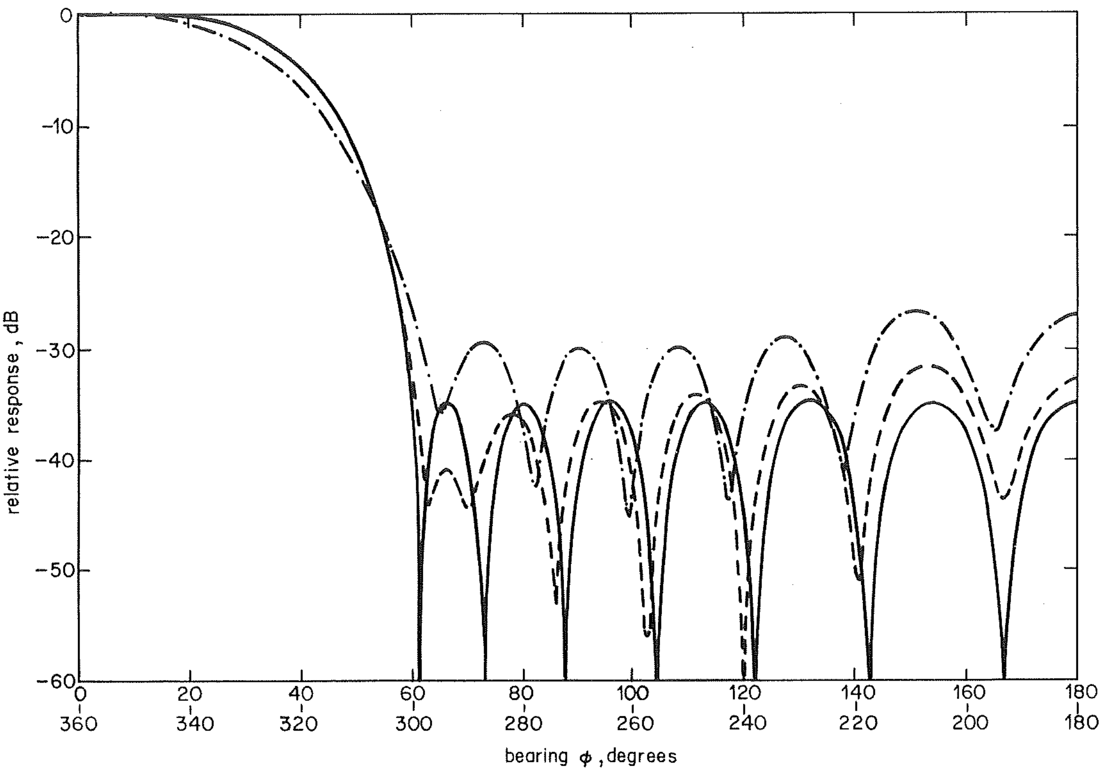


*Fig. 8 - Effect of pre-amplifier input impedance on directivity of eight-dipole array*

$$\frac{\text{Amplifier input resistance}}{\text{Self-impedance of dipole}} = \frac{R}{Z_{11}}$$

——— $R/Z_{11} = \infty$ (Amplifier input conductance = 0)

– – – – $R/Z_{11} = 14{\cdot}3$ (Amplifier input conductance = 1·0 mmho)

—·—·— $R/Z_{11} = 2{\cdot}86$ (Amplifier input conductance = 5·0 mmho)

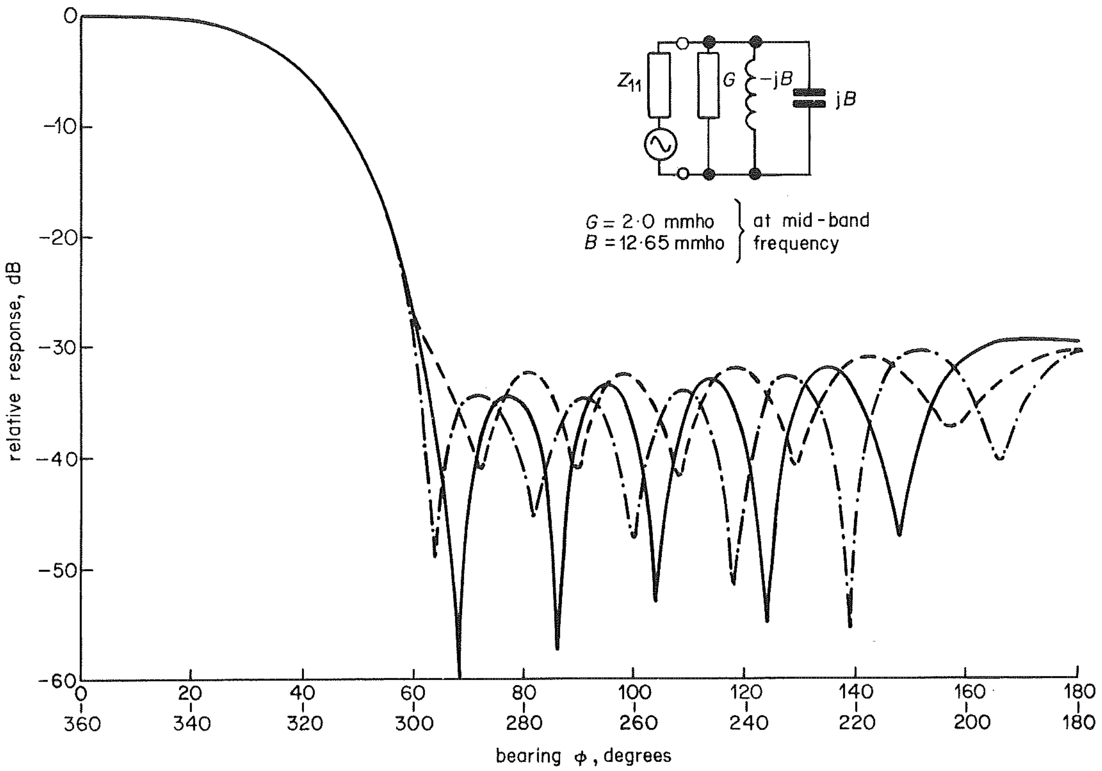


*Fig. 9 - Variation of directivity of an eight-dipole array over 10% bandwidth*

—·—·— 1·05 times mid-band frequency – – – – 0·95 times mid-band frequency

——— Mid-band frequency

9). Say the conductance remains constant over the band of frequencies and that the susceptance is tuned, by shunt inductance ($-jB$ in the insert of Fig. 9), to zero at the mid-band frequency. Suppose that such pre-amplifiers are used in an eight-dipole array similar to that described in Section 3.3. and that each dipole has self impedance (70 + j0) ohms ($Z_{11}$ in the insert of Fig. 9) at the mid-band frequency. The resulting directivities of the array at the mid-band frequency and at the extremes of a 10% bandwidth are shown in Fig. 9, assuming that the gains, $a_r$ (see Section 3.1.), do not vary with frequency. These, and all other quantities involved in the calculations, are given in Appendix 3 (Section 8.3.). It is evident from Fig. 9 that such an array maintains an excellent directional pattern over the 10% bandwidth.

## 4. A PRACTICAL ACTIVE DIPOLE

A practical design for an active dipole is shown in Fig. 10. The choice lay between a simple dipole or a folded dipole. A folded dipole can be made mechanically robust, it has a central earthy point which simplifies mounting, it contains two short-circuited stubs which may readily be used to tune out the input capacitance of the pre-amplifier and other stray capacitance across the dipole terminals ($jB$ in the insert of Fig. 9), it can incorporate its own balun, and it is likely to offer the pre-amplifier greater immunity to nearby lightning strikes (see Section 5). On the other hand a folded dipole requires a pre-amplifier having an input-conductance which is only one quarter of that which would be required for a simple dipole. It has been shown in Section 3.2., however, that this can be achieved with negligible degradation to the noise performance; the latter is particularly true when the effect of galactic noise is considered (see Section 5). A folded dipole was therefore chosen for the foregoing practical reasons.

The dipole shown in Fig. 10, designed for use at 50 MHz, comprises two 1·9 cm diameter tubular aluminium limbs spaced with their centres 10·2 cm apart. The pre-amplifier, having the circuit shown in Fig. 5, is mounted in a cylindrical brass capsule which, as shown in Fig. 10, is connected directly across the dipole terminals by glass-fibre clamps. The output feeder, which also carries the d.c. supply to the pre-amplifier, is enclosed within one of the tubular limbs to form a balun. By making the input conductance of the pre-amplifier 0·5 mmho, and by making the total capacitance across the dipole terminals about 10 pF, the dipole is equivalent to one of those used when calculating the directivities shown in Fig. 9.

The input capacitance of the pre-amplifier, together with the stray capacitance of the glass-fibre clamps, may be tuned out ($-jB$ in the inset of Fig. 9) by suitably positioning the shorting straps on the folded dipole (see Fig. 10). This is not a straightforward adjustment since the straps must be adjusted to tune out this capacitance, independently of any reactance which may be associated with the self-impedance of the dipole ($Z_{11}$ in the insert of Fig. 9). In order to demonstrate not only that this adjustment could be made but also that, when carried out, the re-radiation from the dipole (see Sections 2.1. and 3.1. and the footnote, page 5) is small, the adjustment was made as follows, using active folded dipoles (as shown in Fig. 10) with pre-amplifiers having 0·5 mmho input conductance.

A c.w. signal radiated at 50 MHz was received on an active dipole (to be termed the 'measurement dipole') and its output signal was measured in amplitude and phase relative to a reference signal from the transmitter. A second active dipole (to be termed the 'test dipole') was then mounted parallel to, and spaced 0·2$\lambda$ from, the measurement dipole. Currents flowing in this test dipole as a result of the e.m.f. induced in it by the incident field altered both the amplitude and the phase of the signal from the measurement dipole. This change forms a measure of the re-radiation from the test dipole. The object of the experiment was to demonstrate that, by adjusting the separation between the short-circuiting straps on the test dipole, its re-radiation could be made very small. Accordingly, the signal from the measurement dipole was measured in amplitude and phase, relative to the reference signal from the transmitter, with the test dipole in this position, with various separations between its short-circuiting straps. Taking as unity the output signal from the measurement dipole in the absence of the test dipole, the vector change of output signal caused by the presence of the test dipole will be termed the 'scattering factor.'

The modulus of the resulting scattering factor is shown in Fig. 11 as a function of the strap separation. For completeness, the signal from the measurement dipole was also measured when the test dipole comprised a plain $\lambda/2$ dipole with its central feed-point terminals open-circuited and short-circuited; the resulting scattering factors are also shown in Fig. 11.

It is evident from Fig. 11 that the re-radiation from the active dipole was very small when its shorting straps were spaced 0·175$\lambda$ apart. From the geometry of the dipole it can be shown that this implies a total capacitance of about 9 pF across the dipole terminals. This accords with the value assumed for the calculated directivities shown in Fig. 9.

Attempts were made to demonstrate the principles described in this report by comparing measured and theoretical directivities of an array of four active dipoles but, although good agreement was achieved over the main lobe, agreement over the sidelobes was not good. This was ascribed to the effect of the downleads and the support structure (and also, possibly, the measuring site). Time was not available to investigate further but it is believed that an array of eight or more dipoles, suitably supported at an open site and with the downleads suitably disposed, would give good agreement between theory and practice.

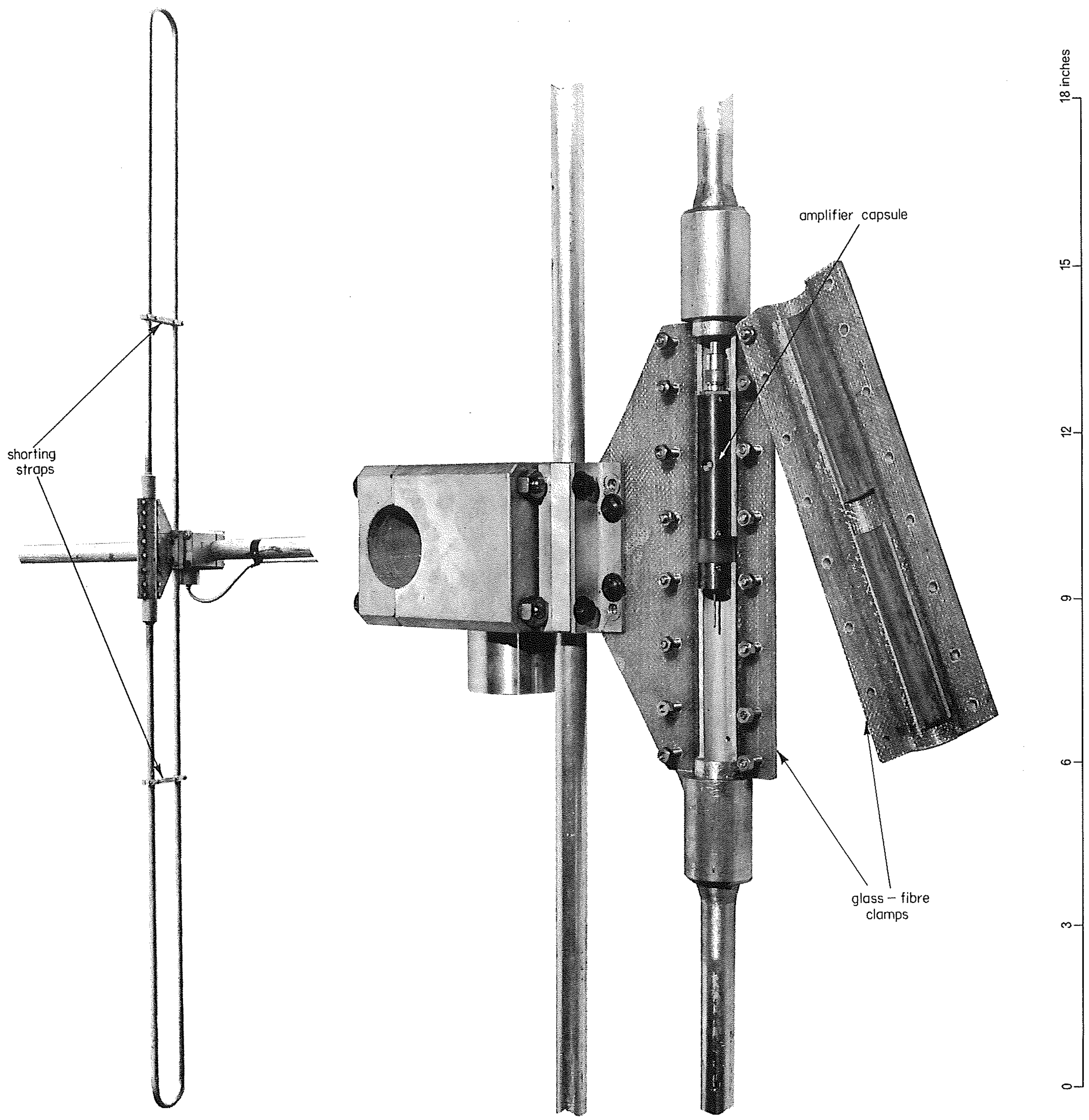


*Fig. 10 • A practical active dipole*

5. FURTHER CONSIDERATIONS

It has been shown in Section 3.2. that pre-amplifiers can be designed to have input impedances up to 2 KΩ with only 1 or 2 dB degradation of noise factor compared to that in the optimally-matched condition. Galactic noise, however, generally dominates thermal noise at v.h.f.; the intrinsic noise temperature of a receiving aerial at 50 MHz, for example, is 8,820°K.[12] Thus if the noise factor of a receiving installation at 50 MHz is worsened from 2 dB to 4 dB (see Fig. 6) the effective noise temperature of the receiving aerial is increased from 8,990°K to 9,260°K. This corresponds to less than 0·15 dB increase in overall noise level.

The effect of tolerances on the amplifier gains ($a_r$) and phases ($r\theta$) – see Section 3.1. – on the directivity must also be considered. This can be done by computing the directivities which result when the values of $a_r$ and $r\theta$ are randomly perturbed between given limits. Time was not available to investigate this fully but a few calculations performed in that way have shown that tolerances of about ± 0·5 dB for the amplifier gains and ± 2° for their phases would probably be sufficient.

The susceptibility of active dipoles to nearby lightning strikes has not been investigated but some form of protection may be necessary. A charged cloud overhead will not cause trouble but, if it is suddenly

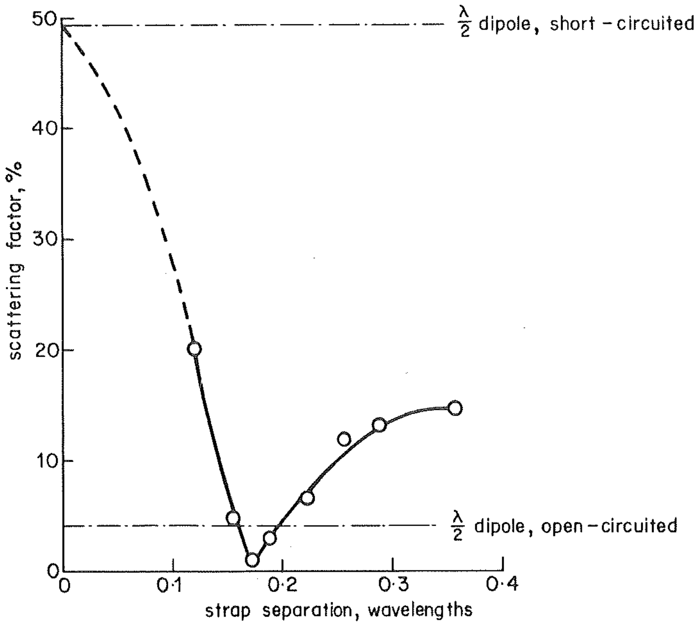


*Fig. 11 - The scattering factor of an active folded dipole as a function of the separation between its shorting straps*

discharged, currents associated with the rapidly collapsing electric field should be prevented from flowing into the pre-amplifiers. Folded dipoles are regarded as superior to plain dipoles in this respect (see Section 4). Alternatively, sufficient protection may be gained from a horizontal earthed wire above the array.

It is clear that the amplifiers must be designed to cause minimum intermodulation but, inevitably, a limiting field strength will exist which, if exceeded, will cause a perceptible level of intermodulation. It may not be possible to provide a high degree of selectivity prior to the pre-amplifiers and, therefore, amplifiers which comfortably handle the wanted signal may be overloaded by a strong signal at a nearby frequency. This must be borne in mind when considering the use of an active r.b.l. aerial at a transmitting station; it could be that such aerials can be used only at receiving sites which are fairly remote from transmitting stations.

The effect of the support structure and of the downleads on the directivity has been mentioned in Section 4. It has not been dealt with in this report but it may be important if high directivities (i.e. sidelobes less than −30 dB) are being considered. Nevertheless other factors (e.g. reflections from sea swell, see Reference 11) may limit the directivity to −30 dB and, if so, reasonable precautions (e.g. the use of wooden support poles with non-conducting stays, and downleads disposed well to the rear of the array) would probably make the disturbance due to downleads and support structure negligible.

## 6. CONCLUSIONS

This report has shown that, by using active dipoles, v.h.f. receiving aerials may be designed which have good noise performance and a directivity which is not impaired by coupling between dipoles. A practical design for an active dipole is given and it is shown that an array of eight such dipoles would maintain, over a 10% bandwidth, a maximum sidelobe level of about −30 dB over 240° range of bearings.

An aerial of this type would be very convenient for re-broadcast television reception at v.h.f. but there are certain problems (discussed in Section 5) which should be considered before such an aerial is adopted.

Aerials of this type are not recommended for re-broadcast reception at u.h.f. because compact highly-directional wide-band u.h.f. receiving aerials can readily be designed using normal techniques. Furthermore, transistors which would enable high-input-impedance amplifiers having good noise performance to be designed are not available at the present time.

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## 8. *APPENDICES*

### 8.1. *APPENDIX 1*

*The Principle of Reciprocity applied to Active Aerials*

The principle of reciprocity may be used to show that the directional characteristics of a given configuration of passive conductors is the same whether it is used for transmission or reception.[10] The following shows how the principle can be extended to active aerials (i.e. aerials comprising any number of conductors, spaced in any manner, for the case where the output from each conductor is connected through an active network to a common combining point).*

Suppose a transmitting array comprises n arbitrarily-spaced conductors (not necessarily similar), each driven between two terminals. Suppose also that the rth conductor is connected through an amplifier to a common pair of input terminals as shown in Fig. 12. Let $a_r$ represent the complex ratio (output e.m.f. at the terminals of the rth conductor)/(voltage at the combining point), thus including both the gain of the rth amplifier and the length of the connections from the common combining point to its input. Let the output impedance of each amplifier be $z$,** let the self-impedance between the terminals of the rth conductor (i.e. the impedance between its terminals when all the other conductors are present but with their terminals open-circuited) be $Z_{rr}$, and the mutual impedance between the rth and mth conductors be $Z_{rm}$. Then if $ea_r$ represents the driving e.m.f. at the output of the rth amplifier, and $i_r$ represents the current flowing into the terminals of the rth conductor, we have:

$$a_1 e = i_1(z + Z_{11}) + i_2 Z_{12} + i_3 Z_{13} + \dots i_n Z_{1n}$$
$$a_2 e = i_1 Z_{21} + i_2(z + Z_{22}) + i_3 Z_{23} + \dots i_n Z_{2n}$$
$$\vdots$$
$$a_n e = i_1 Z_{n1} + i_2 Z_{n2} + i_3 Z_{n3} + \dots i_n(z + Z_{nn})\cdot$$

or, in matrix form,

$$eA = ZI_0 \qquad (5)$$

where A is a column vector having n elements of which the rth is $a_r$, $Z$ is a square matrix having n rows and columns as above, and $I_0$ is a column vector having n elements of which the rth is $i_r$. It follows that:

$$I_0 = eZ^{-1}A \qquad (6)$$

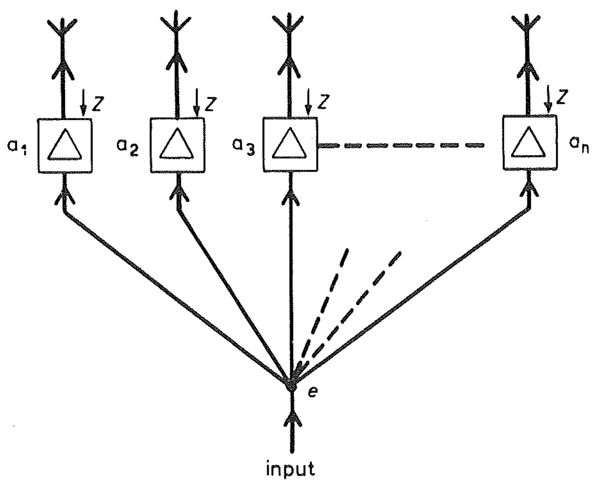


*Fig. 12 - An active transmitting array*

* This proof follows a suggestion by Mr. R.W. Lee.

** For the present purpose, all the amplifiers are considered to have equal output impedances but this is believed to be an unnecessary restriction.

The field, $F_1(\phi)$ at bearing $\phi$, will be proportional to

$$\sum_{r=1}^{n} i_r f_r(\phi),$$

where the n terms $f_r(\phi)$ depend upon the relative position of the n conductors and upon the re-radiation that occurs from them when their terminals are open-circuited.

This can be written

$$F_1(\phi) = k_1 F I_0 \tag{7}$$

where $k_1$ is a (scalar) constant and F is a row vector having n elements of which the rth is $f_r(\phi)$. Hence, combining equations (6) and (7)

$$F_1(\phi) = k_1 e F Z^{-1} A \tag{8}$$

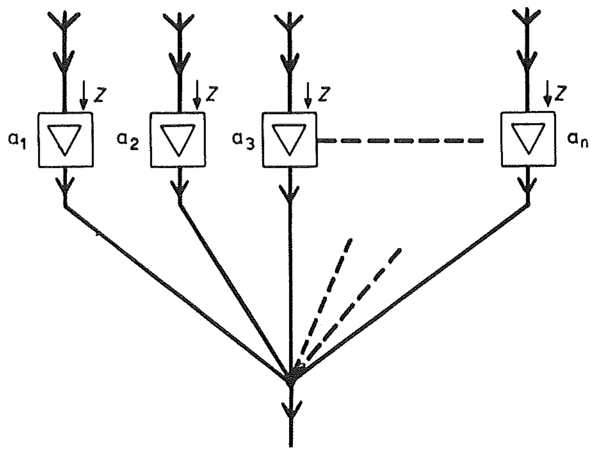


*Fig. 13 - An active receiving array*

For the corresponding system of conductors and amplifiers used, as shown in Fig. 13, as an active receiving array, the terminals of the conductors are directly attached to the amplifier inputs. Each amplifier has an input impedance $z$; the equivalent circuit is shown in Fig. 7 where $\eta f_r(\phi)$ represents the e.m.f. induced in the rth conductor by the field $(\eta)$ incident upon it from a bearing $\phi$, and $i_m Z_{rm}$ represents the e.m.f. induced in the rth conductor by the current $i_m$ flowing in the mth conductor. $\eta$ is the field strength of the incident wave in the absence of the aerial and, as for the transmitting case above, $f_r(\phi)$ is a function which depends upon the relative positions of the n conductors and upon the re-radiation that occurs from them when their terminals are open-circuited.

With the notation used in equation (5)

$$F' = Z I_0 \tag{9}$$

where F' is a column matrix which is the transpose of the row matrix F. It follows that

$$I_0 = Z^{-1} F' \tag{10}$$

The output signal from the complete array (equal to $F_2(\phi)$ for a signal incident from the bearing $\phi$) will be the sum of the contributions from all of the conductors, thus

$$F_2(\phi) = z \sum_{r=1}^{n} a_r i_r = z A' I_0 \tag{11}$$

where A' is a row matrix which is the transpose of the column matrix A. Hence, combining equations (10) and (11)

$$F_2(\phi) = z\,\eta\, A' Z^{-1} F' \tag{12}$$

Now since, by the principle of reciprocity, $Z_{rm} = Z_{mr}$, the matrix $Z$ is symmetric. Thus $Z^{-1} = (Z^{-1})'$ and equation (12) may be written

$$F_2(\phi) = z\,\eta\, A'(Z^{-1})'F'$$
$$= z\,\eta\,(F Z^{-1} A)' \tag{13}$$

Since $F_1(\phi)$ is a scalar quantity (as distinct from a matrix) even though complex, $F_1(\phi) = (F_1(\phi))'$. Thus, combining equations (8) and (13)

$$F_2(\phi) = \frac{z\,\eta}{k_1 e} F_1(\phi) \tag{14}$$

Thus the two directional patterns, $F_1(\phi)$ for the case of transmission and $F_2(\phi)$ for the case of reception, are identical apart from the constant $\frac{z\,\eta}{k_1 e}$

Active receiving arrays of this type therefore have directional characteristics identical to those of similar active transmitting arrays provided that

(a) The active elements are reversed so as to amplify in the reverse direction

and

(b) The output impedances of the active elements for the case of transmission are given a value equal to their input impedances when used for reception.

## 8.2. *APPENDIX 2*

*Quantities Used when Calculating the Results shown in Fig. 8*

*TABLE 1*

*Amplifier gains and phases*

| Element No. (r) | Amplifier gain ($a_r$) | Amplifier phase ($r\theta$) |
|---|---|---|
| 1 | 1·0 | 0 |
| 2 | 2·421 | −144° |
| 3 | 4·090 | −288° |
| 4 | 5·213 | −432° |
| 5 | 5·213 | −576° |
| 6 | 4·090 | −720° |
| 7 | 2·421 | −864° |
| 8 | 1·0 | −1008° |

The element separation ($d$) was 0·4λ

*TABLE 2*

*Self and Mutual Impedances*

| Separation (wavelengths) | Mutual Resistance (ohms) | Mutual Reactance (ohms) |
|---|---|---|
| 0 | 70·0 | 0 |
| 0·4 | +6·5 | −36·9 |
| 0·8 | −19·2 | +11·8 |
| 1·2 | +15·0 | + 1·4 |
| 1·6 | − 7·3 | − 8·7 |
| 2·0 | 0 | + 9·2 |
| 2·4 | + 3·4 | − 7·1 |
| 2·8 | − 6·2 | + 2·6 |

The gains and phases of the amplifiers were as given in Table 1.

The values of self and mutual impedances were as given in Table 2.

The self-impedance is that given by King[13] for a cylindrical dipole 0·00386λ diameter and 0·472λ long in free space. The experimental results of Section 4 show that it is permissible to take this value for the self-impedance of all the dipoles in the array (see Section 8.1). The mutual impedances are those between two infinitely-thin half-wavelength dipoles at the appropriate separation. Whilst these will not strictly apply to a practical array, their values are sufficiently representative for the purpose of Fig. 8.

## 8.3. *APPENDIX 3*

*Quantities used when Calculating the Results shown in Fig. 9*

*TABLE 3*

*Amplifier Gains and Phases at Mid-Band Frequency*

| Element No. | Amplifier Gain ($a_r$) | Amplifier Phase ($r\theta$) |
|---|---|---|
| 1 | 1·0 | 0 |
| 2 | 2·421 | −137 |
| 3 | 4·090 | −274 |
| 4 | 5·213 | −411 |
| 5 | 5·213 | −549 |
| 6 | 4·090 | −686 |
| 7 | 2·421 | −823 |
| 8 | 1·0 | −960 |

The element separation was 0·381λ at the mid-band frequency.

The gains and phases of the amplifiers were, at the mid-band frequency, as given in Table 3. The amplifier gains were assumed not to vary with frequency. The phases were assumed to vary in proportion to the frequency; this corresponds to matched conditions on the inter-connecting cables (see Section 3.1).

The values of self and mutual impedances are given in Table 4. The self impedances are those given by King[13] for a cylindrical dipole which, at the mid-band frequency, is 0·00386λ diameter and 0·472λ long. As for Appendix 2, the mutual impedances are those between two infinitely-thin half-wavelength dipoles at the appropriate separations.

## *TABLE 4*

*Self and Mutual Impedances*

| Separation (wavelengths) at Mid-band frequency | at 0·95 times Mid-band frequency | | at Mid-band frequency | | at 1·05 times Mid-band frequency | |
|---|---|---|---|---|---|---|
| | Mutual Resistance (ohms) | Mutual Reactance (ohms) | Mutual Resistance (ohms) | Mutual Reactance (ohms) | Mutual Resistance (ohms) | Mutual Reactance (ohms) |
| 0 | 82·0 | +38·0 | 70·0 | 0 | 61·0 | −41·5 |
| 0·381 | +6·5 | −36·9 | +10·0 | −37·4 | +14·0 | −37·2 |
| 0·762 | −19·2 | +11·8 | −22·3 | +8·0 | −22·5 | +2·5 |
| 1·143 | +15·0 | +1·4 | +14·2 | +6·5 | +12·0 | +11·0 |
| 1·524 | −7·3 | −8·7 | −3·0 | −11·8 | +2·0 | −12·8 |
| 1·905 | 0 | +9·2 | −5·2 | +9·0 | −9·5 | +5·2 |
| 2·286 | +3·4 | −7·1 | +7·7 | −2·2 | +8·0 | +2·5 |
| 2·667 | −6·2 | +2·6 | −6·5 | −1·8 | −2·2 | −6·2 |

The values of amplifier input admittances used are given in Table 5.

## *TABLE 5*

*Amplifier Input Admittances*

| at 0·95 times Mid-band Frequency | | at Mid-band Frequency | | at 1·05 times Mid-band Frequency | |
|---|---|---|---|---|---|
| Conductance mmho | Susceptance mmho | Conductance mmho | Susceptance mmho | Conductance mmho | Susceptance mmho |
| 2·0 | −1·285 | 2·0 | 0 | 2·0 | +1·345 |

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